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Analysis of Carbon Emission Regulations in Supply Chains with Volatile Demand

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16. Abstract

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Abstract

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Keywords: Carbon emissions, Continuous Review Inventory

1 Introduction

There is a growing consensus that carbon emissions are a leading contributor to global climate change, which has created increasing pressure around the world to enact legislation to curb these emissions. Carbon emission regulations have emerged to address these issues and incentivize firms to curb greenhouse gas (GHG) emissions, primarily carbon-dioxide (other GHG emissions can be measured in terms of carbon-dioxide, see, e.g., EPA, 2013). The US EPA reports that the industrial and commercial sectors contribute 29% and 17%, respectively, to GHG emissions (largely through electricity generation), with transportation adding another 28% (EPA, 2013). Thus, a very large fraction of carbon emissions are due to supply chain activities including inventory holding, freight transportation, and logistics and warehousing activities.

Inventory management is particularly important for a company as this determines not only the level of inventory carried and warehousing activities but also the amount and the frequency of freight shipments and logistical operations. The inventory control policy of a company, therefore, is inextricably linked with its carbon emissions. The objective of this study is to analyze a company's inventory and transportation operations in light of economic and environmental pressures in the presence of demand volatility. Specifically, we capture the tradeoff between traditional economic objectives (cost minimization) and environmental performance (carbon emissions) to allow broader optimization on both dimensions. To this end, this study models and solves a supply chain agent's stochastic inventory control and transportation planning problem under two well-known proposed carbon emission regulation policies: carbon taxing and carbon trading. Under the carbon taxing policy, the supply chain agent is taxed based on its carbon emissions. Under the carbon trading policy, the supply chain agent is subject a carbon cap; however, it can sell or buy carbon allowances through a trading system such as the European Emissions Trading System.

Particularly, our focus is on a retailer's integrated inventory control and supplier selection problem under the aforementioned carbon emission regulation policies. Specifically, we study the continuous review inventory control model under carbon trading policy as the carbon taxing is a special case of the carbon trading policy. In this study, we assume that the retailer can ship his/her order from a set of arbitrary number of suppliers, i.e., multiple suppliers are available. Furthermore, we consider that the retailer can adopt one of the two alternative ordering policies from the suppliers: sequential ordering and sequential delivery. In the former ordering policy, the retailer sequentially orders from the suppliers so that the orders of different suppliers are transported to the retailer's store at the same time. In the later policy, the retailer places the orders from the suppliers are the same time; hence, the orders are transported to the retailer's store sequentially.

2 Literature Review

We note that sustainability has been considered in various operations and supply chain management problems. In this study, our focus is on inventory control studies that consider sustainability. There is a growing body of literature on inventory control models with environmental considerations and we can categorize such studies in two classes: models with deterministic demand and models with stochastic demand. Our study falls in the later class.

2.1 Deterministic Inventory Control Models with Environmental Considerations

Most of the inventory control models with environmental considerations revisit the classic Economic Order Quantity (EOQ) model. The EOQ model analyzes the trade-off between inventory holding and order setup costs for a product that has deterministic demand. Environmental considerations are integrated to the EOQ model either by considering the environmental regulations or by directly associating costs with environmental pollution generated from inventory control related operations or by regarding environmental objectives along with the classical economic objectives.

Particularly, Hua et al. (2011) study the EOQ model under a carbon cap-and-trade policy. They analyze the optimal ordering decisions under such a policy. Furthermore, they provide analytical and numerical results on the effects of the policy parameters, i.e., the carbon cap and the carbon trading price, on the costs and carbon emissions of a company. This study is then extended by Hua et al. (2012) to include pricing decisions for the product. Specifically, it is assumed that the demand for the product is a decreasing function of the market price and the retailer jointly determines the market price and ordering decisions under a carbon cap-and-trade policy. Chen et al. (2013), on the other hand, revisit the EOQ model under a carbon cap policy. They refer to this model as the *carbon-constrained EOQ* and they derive the optimal order quantity decisions for a given cap on carbon emissions. Chen et al. (2013) also investigate in which settings the reduction in carbon emissions is relatively greater than the increase in costs due the carbon cap policy. Furthermore, they discuss what is affecting the difference between the carbon emissions reduction and the cost increase and extend their results for the carbon cap-and-trade and carbon cap-and-offset policies. Arslan and Turkay (2012) formulate and solve the EOQ model under four environmental regulations: carbon cap, carbon tax, carbon cap-and-trade, and carbon cap-and-offset. Their results are similar to those of Hua et al. (2011) and Chen et al. (2013). Toptal et al. (2014) analyze the joint inventory control and carbon emissions reduction investment decisions within the settings of EOQ model under carbon cap, carbon tax, and carbon cap-and-trade policies. They show that these regulation policies will decrease emissions at an expense of increase in costs. Furthermore, they note that availability of carbon emission reduction investment opportunities will enhance the decrease in emissions or lower the increase in costs under any regulation policy compared to the case when carbon emissions abatement investment opportunities are not available. In a recent study, Konur and Schaefer (2014) study the EOQ model with less-than-truckload and truckload carriers under carbon cap, carbon tax, carbon cap-and-trade, and carbon cap-and-offset policies. They note that regulation parameters affect the retailer's choice of carrier and they analytically and numerically investigate the effects of transportation costs and transportation emissions of each carrier on the retailer's total costs and total emissions. Konur (2014) extend the EOQ model with truckload transportation by considering different truck types. Particularly, they analyze this problem under carbon cap policy and discuss that availability of different truck types for transportation might not only reduce costs but also carbon emissions.

While the above studies integrate environmental considerations by modeling the carbon regulation policies within the EOQ model, there are a set of studies that directly associate costs to the environmental pollution generated from the inventory control related operations. Bonney and Jaber (2011), for instance, consider the costs of the emissions generated from the transportation activities and the cost of waste disposal to the environment within the EOQ model. That is, they minimize a cost function that includes the costs associated with inventory holding and order setups as well as the costs of emissions and waste disposal generated from inventory holding and order transportation. Similarly, Ritha and Martin (2012) examine the EOQ model with package costs plus the costs of emissions generated from transportation and packaging operations as well as the costs of waste disposal from the inventory system. Digiesi et al. (2012) also include environmental costs in the EOQ model. Specifically, they mode the cost of emissions generated from transportation and consider the speed of transportation as a decision variable, which also affects the emission generation rate of the transportation operations. In a recent study, Battini et. al (2013) formulate a sustainable EOQ model by defining costs of emissions generated from warehousing activities, inventory holding, and transportation. Particularly, they consider different transportation modes and define emissions cost for each vehicle/container used from each transportation mode for the shipments.

Aside from formulating environmental regulations or associating costs to the environmental damage, environmental considerations are modeled in supply chain management problems by considering environmental objectives as well. For instance, studies by Li et al. (2008), Kim et al. (2009), Ramudhin et al. (2010), Wang et al. (2011), and Chaabane et al. (2012) focus on multi-objective supply chain network design models, where carbon emissions are minimized along with cost minimization or profit maximization objectives. Using a similar approach, Bouchery et al. (2012) define the sustainable EOQ model where the cost as well as a set of sustainability criteria are simultaneously minimized. They focus on determining Pareto efficient order quantity decisions for the proposed sustainable EOQ model. Furthermore, they discuss that sustainability criteria can be reduced with relatively low cost increases.

The aforementioned studies consider inventory control with deterministic demand in a single-echelon setting. We note that inventory control models with deterministic demand have been also studied in two-echelon supply chains. Particularly, the classical vendor-buyer coordination problem with the settings of the EOQ model has been revisited with environmental considerations. For instance, Saadany et al. (2011) analyze the vendor-buyer coordination problem when the demand for the single product depends on its price as well as its environmental quality. The price, environmental quality, and the coordination multiplier are considered as the decision variables of the coordinated channel. Wahab et al. (2011) study a vendor-buyer coordination problem by including emission costs. Similar to Saadany et al. (2011), Zavanella et al. (2012) study the vendor-buyer coordination problem with price- and environmental quality-sensitive demand. They consider environmental quality improvement investment as a decision variable in their model. Jaber et al. (2012), on the other hand, revisit the vendor-buyer coordination model under environmental regulations. Specifically, they consider carbon tax and carbon cap-and-trade regulations in their study. In a recent study, Chan et al. (2013) formulate a multi-objective vendor-buyer coordination problem with single-vendor and multiple buyers. Specifically, they consider incurred cost, wasted energy, wasted raw material, and air pollution as objectives for each buyer.

Finally, it should be noted that inventory control models with deterministic demand other than the classical EOQ models (in single- or multi-echelon channels) have also been analyzed with environmental considerations. For instance, Letmathe and Balakrishnan (2005) study a product mix problem, where the demand for each product depends on the price of the product as well as the emissions generated in manufacturing the product. They consider emissions taxing and trading in their model. Benjaafar et al. (2009) analyze the economic lot sizing problem under carbon cap, tax, cap-and-trade, and offset policies. Absi et al. (2013) investigate the lot sizing problem with different transportation modes under carbon cap policy. They consider two cases for modeling the carbon cap constraint: periodic and cumulative. In the periodic case, carbon emissions are restricted for each period. On the other hand, in the cumulative case, total carbon emissions generated until the end of the planning period are restricted by the carbon cap. In a recent study, Palak et al. (2014) study economic lot sizing problem with multiple suppliers under carbon cap, carbon tax, carbon cap-and-trade, and carbon cap-and-offset policies.

2.2 Stochastic Inventory Control Models with Environmental Considerations

Compared to the studies on inventory control models with environmental considerations under deterministic demand, the studies assuming stochastic demand are rather limited. Particularly, the single-period stochastic demand inventory model, i.e., the classical newsvendor model, has been revisited with environmental considerations. Song and Leng (2012) study the newsvendor problem under carbon cap, tax, and cap-and-trade policies. Similarly, Liu et al. (2013) revisit the newsvendor problem with carbon cap-and-trade mechanism. They discuss the effects of policy parameters on the ordering decisions. Zhang and Xu (2013) extend the single-item newsvendor model to the multi-item newsvendor model and analyze the multi-item newsvendor problem under a cap-and-trade mechanism. They provide analytical results on the effects of carbon trading price on ordering decisions. In the settings of the newsvendor model, Hoen et al. (2013) first study the transportation mode selection problem with carbon emissions costs. They analyze the effects of emission costs on the solution of the problem. Then, they provide numerical examples for the newsvendor model with transportation mode selection under carbon cap, tax, and cap-and-trade policies. Rosic and Jammernegg (2013) also focuses on newsvendor problem and they study the effects of dual sourcing on costs and carbon emissions.

In a recent study, Arikan et al. (2013) consider carbon emissions in a continuous review inventory control model. Specifically, while they do not mathematically formulate a (Q, R) model with environmental considerations, they do a simulation study to illustrate the effects of demand variability on carbon emissions generated under different transportation modes. To the best knowledge of the authors, the continuous inventory control problem with supplier selection has not been studied under carbon regulations in the literature. This paper intends to close this gap and discuss the effects of different ordering policies on a retailer's costs and carbon emissions.

3 Problem Formulation

We consider a retailer's inventory control problem for a single item which has stochastic demand. Let the demand per unit time for the item be a normally distributed random variable with mean λ and standard deviation υ. We, therefore, assume that the demand during a time period of t is normally distributed with mean λt and standard deviation $\upsilon\sqrt{t}$ (see, e.g., Nahmias, 2008). Due to stochastic demand, there might be shortages and let $n(r,t)$ be the expected number of shortages over a time period t when the starting inventory is r. It then follows that $n(r,t) = \int_r^\infty (x-r)f(x)dx$ where $f(x)$ is the normal density function with mean λt and standard deviation $\upsilon\sqrt{t}$. Note that $\lim_{t\to 0} n(r,0) = 0$. It is assumed that the inventory is continuously reviewed, i.e., the retailer knows the inventory level at any moment. In case of continuous inventory review, a common inventory control policy adopted is (Q, R) model, where Q denotes the order quantity and R denotes the re-order point to place an order. That is, whenever the inventory on hand is R, an order of Q units is placed.

In the settings of the classical (Q, R) model, the retailer is subject to inventory holding, penalty, procurement, and order setup costs. Let $\widetilde{h}$ denote the retailer's per unit per unit time inventory holding cost. It is assumed that all of the shortages are backordered and there is a penalty cost $\widetilde{p}$ per unit backordered. In this study, we assume that the retailer can partially order his/her order quantity from a set of n suppliers, indexed by i such that $i = \{1, 2, \ldots, n\}$, i.e., we allow order splitting. As different suppliers might have distinct characteristics with regards to their locations, wholesale prices, and shipment requirements, we define $\widetilde{c}_i$ and $\widetilde{a}_i$ as the retailer's unit procurement and fixed order setup cost from supplier i, respectively. For instance, $\widetilde{c}_i$ can be defined to include the supplier's unit transportation cost as well and $\widetilde{a}_i$ can include the fixed transportation or delivery cost such as the truck driver's cost or loading/unloading charges. Furthermore, we assume that each supplier has a shipment capacity of w_i units per order due to limited supply or the capacity of the transportation mode used by supplier i; and, we define τ_i as the delivery lead time of supplier i and it is assumed that different suppliers might have different lead times due to different points of origin or transportation modes used.

As noted in Section 1, there is a significant amount of carbon emissions generated from inventory holding, freight transportation, and warehousing activities. Similar to Hua et al. (2011), Chen et al. (2013), and Toptal et al. (2014), we assume that $\widehat{h}$ units of carbon emissions generated from holding one unit inventory per unit time due to electricity used in the warehouse for cooling/heating/lighting operations. We also consider that $\widehat{p}$ units of carbon emissions are generated from backordered shortages as the retailer might need to ship the backordered unit to the customer (see, e.g., Anderson et al., 2012) or the customer might need to re-travel to the retailer's store to pick the backordered unit (see, e.g., Cachon, 2014). A substantial amount of carbon emissions are due to freight transportation and the transportation emissions depend on the transportation mode selected, type of vehicles used, the load carried, and the shipment distance (Konur, 2014, Konur and Schaefer, 2014). As different suppliers can use different transportation modes, or even different vehicle types of the same transportation mode, we consider that each supplier's delivery has different carbon emissions generation characteristics. In particular, we let $\widehat{c}_i$ be the carbon emissions generated per unit shipped by supplier i and $\widehat{a}_i$ denote the fixed carbon emissions generated per shipment made by supplier i. For instance, $\widehat{a}_i$ can be considered as the carbon emissions generated due to the empty weight of the transportation unit (e.g., a truck) and $\widehat{c}_i$ is the carbon emissions generated from each additional load (similar parameters are defined in Hua et al., 2011, Chen et al., 2013, Konur, 2014).

In this study, we assume that the retailer is subject to one of the two most-common environmental regulations: carbon taxing and carbon trading. Under carbon taxing, the retailer is charged per unit of carbon emissions generated and let α denote the carbon tax per unit of carbon emissions generated. On the other hand, under carbon trading, the retailer is subject to a carbon emissions limit per unit time, known as carbon cap, and carbon emissions are tradable. Particularly, if the retailer's carbon emissions per unit time is below the carbon cap, the retailer can sell his/her excess carbon emissions; on the other hand, if the retailer's carbon emissions per unit time is above the carbon cap, the retailer needs to buy the extra carbon allowances. Let β denote the carbon trading price per unit of carbon emissions and Φ be the carbon cap per unit time. Similar to Hua et al. (2011) and Toptal et al. (2014), we assume that there is sufficient demand and supply for carbon trading in the market; hence, the retailer can sell all of his excess carbon allowances or buy unlimited carbon allowances. One can note that when $\Phi = 0$, carbon taxing and carbon trading regulations are identical if $\beta = \alpha$. Therefore, in the mathematical formulation and the solution analysis, we will only focus on carbon trading regulation as carbon taxing is a special case of carbon trading.

The retailer's objective is to minimize his/her total expected costs per unit time by determining which suppliers to select, how much to ship from each supplier, and when to start ordering from the suppliers. Let

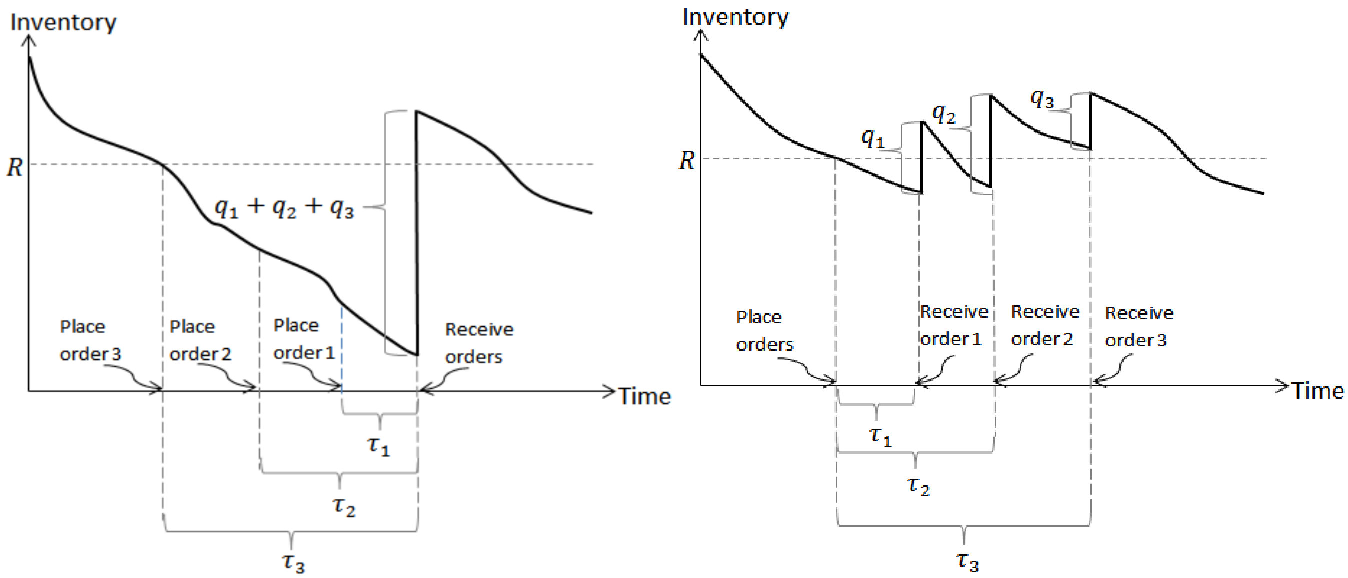
$$x_i = \begin{cases} 1 & \text{supplier } i \text{ is selected,} \\ 0 & \text{otherwise} \end{cases}$$

and $\mathbf{x}$ be the binary n-vector of x_i values. Furthermore, let q_i be the quantity ordered from supplier i at each replenishment and $\mathbf{q}$ denote the n-vector of q_i values. Note that if $x_i = 0$ then $q_i = 0$ and if $x_i = 1$ then $q_i \leq w_i$. As is defined previously, R is the re-order point.

We assume that the supplier can use one of the two policies for order splitting among the selected suppliers: sequential ordering and sequential delivery. In sequential ordering, the retailer places the orders from different suppliers sequentially considering their lead times such that the orders from different suppliers are received by the retailer at the same time. In sequential delivery, the retailer places the orders from different supplier at the same time and the orders from different suppliers are received by the retailer at different times due to different lead times. Figure 1 illustrates the retailer's inventory over time when three suppliers such that $\tau_1 < \tau_2 < \tau_3$ are used with sequential ordering and sequential delivery. In sequential delivery, we assume that the next orders will not be placed until the order of the last supplier has been delivered. In what follows, we mathematically formulate the retailer's inventory control and supplier selection problem with each order splitting policy.

Figure 1: Inventory vs. Time

(a) Sequential Ordering Policy (b) Sequential Delivery Policy



3.1 Sequential Ordering

In the case the retailer adopts sequential ordering policy, the effective lead time, i.e., the time between the retailer starts ordering from the suppliers until the orders are simultaneously received, is the maximum of the lead times of the selected suppliers. Let $\tau(\mathbf{x})$ denote the effective lead time when supplier selection decision is given by $\mathbf{x}$. It then follows that

$$\tau(\mathbf{x}) = \max_i \{\tau_i x_i\}. \tag{1}$$

The expected inventory level under sequential ordering is defined similar to the classical (Q, R) model and one can derive that $\widetilde{h}\left(R - \lambda\tau(\mathbf{x}) + \frac{1}{2}\sum_{i=1}^n q_i\right)$ is the expected inventory holding cost per unit time. Similarly, it can be argued that the expected cycle length (the time between receiving two consecutive the orders from the suppliers) is $\frac{1}{\lambda}\sum_{i=1}^n q_i$; thus, the expected procurement cost per unit time and the expected order setup cost per unit time amount to $\frac{\lambda\sum_{i=1}^n \widetilde{c}_i q_i}{\sum_{i=1}^n q_i}$ and $\frac{\lambda\sum_{i=1}^n \widetilde{a}_i x_i}{\sum_{i=1}^n q_i}$, respectively. Finally, shortages can occur during the effective lead time and the expected number of shortages per cycle is $n(R, \tau(\mathbf{x}))$. It then follows that the expected penalty cost per unit time is equal to $\frac{\widetilde{p}\lambda n(R,\tau(\mathbf{x}))}{\sum_{i=1}^n q_i}$. These imply that the retailer's expected total cost per unit time under sequential ordering as a function of the decision variables R, $\mathbf{q}$, and $\mathbf{x}$, denoted by $C^1(R, \mathbf{q}, \mathbf{x})$, is

$$C^1(R, \mathbf{q}, \mathbf{x}) = \frac{\lambda \sum_{i=1}^{n} \widetilde{c}_i q_i}{\sum_{i=1}^{n} q_i} + \widetilde{h}\left(R - \lambda\tau(\mathbf{x}) + \frac{1}{2}\sum_{i=1}^{n} q_i\right) + \frac{\lambda \sum_{i=1}^{n} \widetilde{a}_i x_i}{\sum_{i=1}^{n} q_i} + \frac{\widetilde{p}\lambda n(R, \tau(\mathbf{x}))}{\sum_{i=1}^{n} q_i}, \tag{2}$$

where the first, second, third, and the last terms are the expected procurement, inventory holding, order setup, and penalty cost per unit time, respectively, such that $\tau(\mathbf{x})$ is defined in Equation (1).

The expected carbon emissions generated from inventory related operations can be defined similar to the expected inventory related costs given in Equation (2). Particularly, it can be shown that the retailer's carbon emissions per unit time under sequential ordering as a function of the decision variables R, $\mathbf{q}$, and $\mathbf{x}$, denoted by $E^1(R, \mathbf{q}, \mathbf{x})$, reads

$$E^1(R, \mathbf{q}, \mathbf{x}) = \frac{\lambda \sum_{i=1}^{n} \widehat{c}_i q_i}{\sum_{i=1}^{n} q_i} + \widehat{h}\left(R - \lambda\tau(\mathbf{x}) + \frac{1}{2}\sum_{i=1}^{n} q_i\right) + \frac{\lambda \sum_{i=1}^{n} \widehat{a}_i x_i}{\sum_{i=1}^{n} q_i} + \frac{\widehat{p}\lambda n(R, \tau(\mathbf{x}))}{\sum_{i=1}^{n} q_i}, \tag{3}$$

where the first, second, third, and the last terms are the expected carbon emissions generated per unit time from transportation, inventory holding, order setup, and backordering operations, respectively, such that $\tau(\mathbf{x})$ is defined in Equation (1).

Under a carbon trading policy with carbon cap Φ, the total amount of traded carbon emissions is equal to $E^1(R, \mathbf{q}, \mathbf{x}) - \Phi$. Note that if $E^1(R, \mathbf{q}, \mathbf{x}) - \Phi > 0$, the retailer is buying extra carbon allowances at a cost of β; and, if $E^1(R, \mathbf{q}, \mathbf{x}) - \Phi < 0$, the retailer is selling his/her excess carbon emissions at a price of β. The retailer's optimization problem under carbon trading with sequential ordering policy then can be formulated as follows:

$$\begin{array}{lll} (P1): & \min & F^1(R, \mathbf{q}, \mathbf{x}) = C^1(R, \mathbf{q}, \mathbf{x}) + \beta(E^1(R, \mathbf{q}, \mathbf{x}) - \Phi) \\ & \text{s.t.} & 0 \leq q_i \leq x_i w_i \quad \forall\, i \\ & & x_i \in \{0, 1\} \quad \forall\, i \\ & & R > 0. \end{array}$$

$F^1(R, \mathbf{q}, \mathbf{x})$ defines the total costs per unit time and the first set of constraints guarantees that the retailer can only order from the selected suppliers and the order quantity from each selected supplier is less than or equal to the supplier's capacity. The second constraint is the binary definition of x_i values and the third constraint is the non-negativity of the re-order point. Let R^1, $\mathbf{q}^1$, and $\mathbf{x}^1$ denote the optimum solution of P1.

3.2 Sequential Delivery

In the case the retailer adopts sequential delivery policy, we define a cycle as the time between receiving two consecutive orders from the same supplier; therefore, the expected cycle length can be defined similar to the classical (Q, R) model. That is, the expected cycle length is $\frac{1}{\lambda}\sum_{i=1}^{n} q_i$. It then follows that the expected procurement cost per unit time is equal to $\frac{\lambda \sum_{i=1}^{n} \widetilde{c}_i q_i}{\sum_{i=1}^{n} q_i}$ and the expected order setup cost per unit time is equal to $\frac{\lambda \sum_{i=1}^{n} \widetilde{a}_i x_i}{\sum_{i=1}^{n} q_i}$. Defining the expected inventory holding cost and expected penalty cost per unit time, on the other hand, is different that the sequential ordering policy. To do so, without loss of generality, let us assume that the suppliers are sorted such that $\tau_1 < \tau_2 < \ldots < \tau_n$. Furthermore, let $\tau_0 = 0$ and $\tau_{n+1} = \frac{1}{\lambda}\sum_{i=1}^{n} q_i$. Note that τ_{n+1} defines the expected cycle length.

Given that $x_i = 1$ for $i \leq k$ and $x_i = 0$ for $i \geq k+1$ such that $k+1 \leq n$, one can show that the expected inventory held during one cycle amounts to $R + \sum_{i=1}^{k} q_i(\tau_{n+1} - \tau_i) - \frac{\lambda}{2}\tau_{n+1}^2$, which does not depend on x_i values. It then can be concluded that the expected inventory holding cost per unit time for any $\mathbf{x}$ is equal to $h\left(R - \frac{\lambda \sum_{i=1}^{n} \tau_i q_i}{\sum_{i=1}^{n} q_i} + \frac{1}{2}\sum_{i=1}^{n} q_i\right)$. Notice that we will guarantee that $q_i = 0$ if $x_i = 0$ by adding constraints in formulating the retailer's optimization problem.

Now, let us focus on defining the expected penalty cost per unit time. To do so, we first calculate the expected number of shortages within one cycle. The number of shortages can occur during the time periods from the moment orders placed and until the first order received, from the moment first order received until the second order received, and so on. For instance, suppose that $x_i = 1$ for $i \leq k$ and $x_i = 0$ for $i \geq k+1$ such that $k+1 \leq n$. In this case, the shortages can occur during the following k time periods $[t+\tau_0, t+\tau_1), [t+\tau_1, t+\tau_2), \ldots, [t+\tau_{k-1}, t+\tau_k)$, where t is the moment the orders are placed (recall that $\tau_0 = 0$). We refer to these time periods as shortage periods. Note that the length of the shortage periods are $\tau_1 - \tau_0, \tau_2 - \tau_1, \ldots, \tau_k - \tau_{k-1}$, respectively, and one can show that the expected inventory at the beginning of the shortage periods are $R, R - \lambda\tau_1 + q_1, R - \lambda\tau_2 + q_1 + q_2, \ldots, R - \lambda\tau_{k-1} + \sum_{i=1}^{k-1} q_i$, respectively. Following this discussion, for any $\mathbf{x}$, the length of the i^{th} shortage period, i.e., the time between receiving the orders from $(i-1)^{th}$ and i^{th} suppliers, denoted by $t_i(\mathbf{x})$, is equal to

$$t_i(\mathbf{x}) = \max\left\{0, \tau_i x_i - \max_{j:0 \leq j \leq (i-1)}\{\tau_j x_j\}\right\}. \tag{4}$$

Equation (4) implies that $t_i(\mathbf{x}) = 0$ when $x_i = 0$; hence, no shortages will occur. Particularly, when $x_i = 1$, $t_i(\mathbf{x})$ defines a positive time length of $t_i - t_j$ where j is the supplier, which is selected, with the highest lead time that has shorter lead time than supplier i. Similarly, one can define the starting inventory at the i^{th} shortage period, denoted by $r_i(\mathbf{q}, \mathbf{x})$ as

$$r_i(\mathbf{q}) = R - \lambda\tau_{i-1} + \sum_{j=1}^{i-1} q_j. \tag{5}$$

It should be remarked that $r_i(\mathbf{q})$ defined in Equation (5) is not necessarily zero even if $x_i = 0$; however, this will not affect the expected number of shortages during the i^{th} shortage period as the length of the i^{th} shortage period is 0 from Equation (4) and $n(r,t) = 0$ when $t = 0$. Therefore, the expected number of shortages within one cycle amounts to $\sum_{i=1}^{n} n(r_i(\mathbf{q}), t_i(\mathbf{x}))$.

The above discussion leads that the retailer's expected total cost per unit time under sequential delivery as a function of the decision variables R, $\mathbf{q}$, and $\mathbf{x}$, denoted by $C^2(R,\mathbf{q},\mathbf{x})$, is

$$C^2(R,\mathbf{q},\mathbf{x}) = \frac{\lambda \sum_{i=1}^{n} \widetilde{c}_i q_i}{\sum_{i=1}^{n} q_i} + \widetilde{h}\left(R - \frac{\lambda \sum_{i=1}^{n} \tau_i q_i}{\sum_{i=1}^{n} q_i} + \frac{1}{2}\sum_{i=1}^{n} q_i \right) + \frac{\lambda \sum_{i=1}^{n} \widetilde{a}_i x_i}{\sum_{i=1}^{n} q_i} + \frac{\widetilde{p}\lambda \sum_{i=1}^{n} n(r_i(\mathbf{q}), t_i(\mathbf{x}))}{\sum_{i=1}^{n} q_i}, \tag{6}$$

where the first, second, third, and the last terms are the expected procurement, inventory holding, order setup, and penalty cost per unit time, respectively, such that $t_i(\mathbf{x})$ and $r_i(\mathbf{q})$ are defined in Equations (4) and (5), respectively.

The expected carbon emissions generated from inventory related operations can be defined similar to the expected inventory related costs given in Equation (6). Particularly, it can be shown that the retailer's carbon emissions per unit time under sequential delivery as a function of the decision variables R, $\mathbf{q}$, and $\mathbf{x}$, denoted by $E^2(R,\mathbf{q},\mathbf{x})$, reads

$$E^2(R,\mathbf{q},\mathbf{x}) = \frac{\lambda \sum_{i=1}^{n} \widehat{c}_i q_i}{\sum_{i=1}^{n} q_i} + \widehat{h}\left(R - \frac{\lambda \sum_{i=1}^{n} \tau_i q_i}{\sum_{i=1}^{n} q_i} + \frac{1}{2}\sum_{i=1}^{n} q_i \right) + \frac{\lambda \sum_{i=1}^{n} \widehat{a}_i x_i}{\sum_{i=1}^{n} q_i} + \frac{\widehat{p}\lambda \sum_{i=1}^{n} n(r_i(\mathbf{q}), t_i(\mathbf{x}))}{\sum_{i=1}^{n} q_i}, \tag{7}$$

where the first, second, third, and the last terms are the expected carbon emissions generated per unit time from transportation, inventory holding, order setup, and backordering operations, respectively, such that $t_i(\mathbf{x})$ and $r_i(\mathbf{q})$ are defined in Equations (4) and (5), respectively.

Similar to P1, the retailer's optimization problem under carbon trading with sequential delivery policy can be formulated as follows:

$$\begin{array}{lll} (P2): & \min & F^2(R,\mathbf{q},\mathbf{x}) = C^2(R,\mathbf{q},\mathbf{x}) + \beta(E^2(R,\mathbf{q},\mathbf{x}) - \Phi) \\ & \text{s.t.} & 0 \le q_i \le x_i w_i \quad \forall\ i \\ & & x_i \in \{0,1\} \quad \forall\ i \\ & & R > 0. \end{array}$$

Let R^2, $\mathbf{q}^2$, and $\mathbf{x}^2$ denote the optimum solution of P2.

4 Solution Analysis

We note that both P1 and P2 are mixed-integer-nonlinear optimization problems. Therefore, in this section, we focus on developing heuristic methods to determine $(R^1, \mathbf{q}^1, \mathbf{x}^1)$ and $(R^2, \mathbf{q}^2, \mathbf{x}^2)$. Particularly, we construct a greedy neighbor search method for both of the models. Starting from a solution, the greedy neighbor search method searches for better solutions in its neighborhood. If there is a better solution in the current neighborhood, the method moves to the best neighbor and starts searching for a better solution in the new neighborhood. If there is no better solution in the neighborhood, the search stops. Next, we discuss the details of the greedy neighbor search method (GNSM).

To construct GNSM, a solution should be defined such that a finite neighborhood can be defined. In models P1 and P2, the binary vector $\mathbf{x}$ is the perfect candidate to define a solution. Specifically, suppose that $\mathbf{x} = \mathbf{x}^0$ is given. Then $\mathbf{x}^0$ will have n neighbors and let $\mathbf{x}^{0i}$ be the i^{th} neighbor of $\mathbf{x}^0$ such that $x_j^0 = x_j^{0i}$ for $j \neq i$ and $x_i^{0i} = 1 - x_i^0$. To see whether there exists a better solution in the neighborhood of $\mathbf{x}^0$, one needs to calculate the minimum total expected costs per unit time given $\mathbf{x}^0$. Let $R^m(\mathbf{x}^0)$ and $\mathbf{q}^m(\mathbf{x}^0)$ be the minimizer of $F^m(R, \mathbf{q}, \mathbf{x}|\mathbf{x} = \mathbf{x}^0)$ for $m = 1, 2$. Note that $F^m(R, \mathbf{q}, \mathbf{x}|\mathbf{x} = \mathbf{x}^0)$ is a non-linear function with upper limits on q_i values. Furthermore, convexity of $F^m(R, \mathbf{q}, \mathbf{x}|\mathbf{x} = \mathbf{x}^0)$ is not guaranteed. Interior point method is generally preferred for minimization of nonlinear functions; therefore, we use interior point method to determine $R^m(\mathbf{x}^0)$ and $\mathbf{q}^m(\mathbf{x}^0)$. Then, GNSM works as follows:

1. Let $\mathbf{x}^0$ be given and set $\mathbf{x}^* = \mathbf{x}^0$.
2. Determine $R^m(\mathbf{x}^0)$ and $\mathbf{q}^m(\mathbf{x}^0)$ using interior point method and set $R^* = R^m(\mathbf{x}^0)$ and $\mathbf{q}^* = \mathbf{q}^m(\mathbf{x}^0)$
3. For $i = 1 : n$
   (a) Let $\mathbf{x}^{0i} = \mathbf{x}^0$ and set $x_i^{0i} := 1 - x_i^0$
   (b) Determine $R^m(\mathbf{x}^{0i})$ and $\mathbf{q}^m(\mathbf{x}^{0i})$ using interior point method
4. End
5. If $F^m(R^m(\mathbf{x}^0), \mathbf{q}^m(\mathbf{x}^0), \mathbf{x}^0) \geq \min_i\{F^m(R^m(\mathbf{x}^{0i}), \mathbf{q}^m(\mathbf{x}^{0i}), \mathbf{x}^{0i})\}$
   (a) Let $\mathbf{x}^0 = \arg\min_i\{F^m(R^m(\mathbf{x}^{0i}), \mathbf{q}^m(\mathbf{x}^{0i}), \mathbf{x}^{0i})\}$
   (b) Set $R^* = R^m(\mathbf{x}^0)$, $\mathbf{q}^* = \mathbf{q}^m(\mathbf{x}^0)$, and $\mathbf{x}^* = \mathbf{x}^0$ and go to Step 3.
6. Else
   (a) Stop and return R^*, $\mathbf{q}^*$, and $\mathbf{x}^*$.

Note that GNSM is guaranteed to find a local optimum solution. To avoid that GNSM gets stuck with a bad local optimum solution, we run GNSM with n different starting solutions such that each starting solution corresponds to a supplier selection with a single supplier. The best starting solution is the retailer's single-sourcing solution, i.e., when the retailer can order only from a single supplier.

5 Future Work

In the rest of this project, numerical analysis will be conducted for the following analysis:

- Effects of carbon tax, carbon trading price, and carbon cap on the retailer's costs and carbon emissions with sequential ordering and sequential delivery,
- Effects of demand variability on the retailer's costs and carbon emissions with sequential ordering and sequential delivery,
- Comparison of single sourcing, sequential ordering, and sequential delivery policies with respect to the retailer's costs and carbon emissions.